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A Method to Determine the Velocity Profiles from the Power Consumption of Electric Vehicles

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Abstract

Acceleration and deceleration are the most important features in performance assessments concerning car's performance. Whether fuel consumption is good or bad has dominated the performance of cars in recent years, and thus many electric vehicles (EV) with high energy-efficiency have been developed. But problems relating to the running distance per charging, the charging time, and so on of electric vehicles have been preventing them from spreading more generally. Under this circumstance, we will analyze the acceleration and deceleration performances of cars and examine the most suitable method for using energy effectively. As to an EV, deriving the accelerated velocity, a, from a situation when electric input power, P, is given to an EV, driving at a velocity, v, is extremely important in order to find out the acceleration performance of a car. This paper analytically describes the results derived from the relationship between obtained the acceleration, a, and the input or regenerated power, P,. Also, this paper shows that the results obtained from the profile of the predicted velocity, v, based on the input electric-power, P, of accelerating cars, corresponds to the characteristics actually measured. It is pointed out here that the EV is very convenient to perform this analysis, because the value of the eclectic input power can be obtained easily by a current and the voltage to the motor on board. This method demonstrates possibilities for predicting the performance of electric vehicles.

Keywords

electric vehicle, velocity profile, battery, power consumption, theoretical prediction

1. INTRODUCTION

Predicting the velocity actually obtained from the applied power to vehicles is a very important factor in design. But it is not easy to get the generated horse power as a measured value in the case of internal-combustion vehicles. In the case of electric vehicles, on the other hand, it is possible to know the generated power by simply measuring the voltage and the current of the electric power supply such as a battery system. This paper shows a formula for the relationship between electric input power and the velocity profile of electric vehicles. Furthermore, it is also shown that, with the results, accelerated velocity is calculated by the applied time variation of input power, P, in the case of a running EV, and then the velocity variation of the EV is obtained. Needless to say, a running-resistance parameter for electric vehicles is needed. [Chan et al., 2001; Nasukawa et al., 1993]

2. EV ACCELERATION THEORY

Hypothetically, when an electric vehicle with a velocity, v [m/s], is running with input electricity, P [W], to its motor, the acceleration, a [m/s^2], is obtained in a very brief time of, Dt, seconds. In this case, the formula for energy conservation is shown in the following equation with motor efficiency, η, .

$$\eta P \cdot \Delta t = v(kv^2 + L) \cdot \Delta t + \frac{1}{2} m[(v + a \cdot \Delta t)^2 - v^2]$$
$$= v(kv^2 + L) \cdot \Delta t + \frac{1}{2} m(2av \cdot \Delta t + a^2 \Delta t^2)$$

And then, as $\Delta t^2 = 0$,

$$\eta P = v(ma + kv^2 + L) \tag{1}$$

Here, kv^2, is the air-resistance coefficient, L, is the running resistance which doesn't relate to the velocity but corresponds to rolling-resistance and inclination resistance.

Next, the efficiency, η, should be considered. In the case of a DC motor, losses are mainly copper loss for the high-current region. Thus, the efficiency, η, is expressed by the following equation, according to the efficiency definition.

$$\eta = \frac{P - I^2 r}{P}$$

Here, I, represents the current and, r, represents coil resistance. In the case of an AC motor, r, is used as equivalent values including iron loss. When this is assigned to Eq. (1), it becomes:

$$P = I^2 r + v(ma + kv^2 + L) \qquad (2)$$

This shows that the input power is converted to energy of the copper loss, driving, and acceleration of the motor. The following equation shows the relationship of a DC motor's back-electromotive force, E, current, I, velocity of vehicle, v, and force, F, .

$$E = \frac{Ka\phi N}{R} v = Kv \qquad (3)$$

$$F = \frac{Ka\phi N}{R} I = KI \qquad (4)$$

Here, Ka, represents the DC motor innate armature-constant, φ, is the magnetic flux on the armature, R, is the radius of the tire and, N, is the reduction ratio caused by gears and others.

When assigning equation (4) to (2),

$$P = \frac{r}{K^2} F^2 + v(ma + kv^2 + L)$$

In this case, F, is shown as the following equation, according to the motion equation.

$$F = ma + kv^2 + L$$

When this is assigned, Eq. (2) becomes,

$$P = \frac{r}{K^2}(ma + kv^2 + L)^2 + v(ma + kv^2 + L) \qquad (5)$$

This equation shows the acceleration, a, when input power, P, is given to a vehicle running at a certain velocity, v, . Eq. (5) can be changed to the following equation, which is used to calculate the accelerated velocity from the input power to the motor

$$a = \frac{+K\sqrt{v^2K^2 + 4rP} - (2krv^2 + 2rL + vK^2)}{2mr} \qquad (6)$$

Based on Eq. (6), when the velocity and the input power at a certain time are given, the accelerated velocity at the time can be obtained. This equation has a positive and negative code (±) . The (+) indicates the forward driving and (–) indicates the reverse driving.

Eq. (6) allows a condition of input power, $P<0$, . This negative electricity is a regenerating energy towards a power-supply unit. According to the condition whereby an inside radical sign is positive, the following equation is obtained.

$$P > -\frac{v^2K^2}{4r} \qquad (7)$$

In short, the energy range, in which regenerating is possible at a velocity, v, is $0 > P \geq -\frac{v^2K^2}{4r}$, and the accelerated velocity is the minimum under the condition of Eq. (7).

3. REQUIRED INPUT POWER

Eq. (5) is transformed,

$$P = \frac{m^2 r}{K^2}[a + \frac{1}{2m}(2kv^2 + 2L + \frac{vK^2}{r})]^2 - \frac{v^2K^2}{4r} \qquad (8)$$

The electricity, P, needed to obtain an acceleration velocity, a, can be calculated from this equation. According to Eq. (7), the necessary electricity, P, is convex to downward in relation to the acceleration velocity, a, . And in the case of Eq. (8), it becomes the minimum.

$$a = -\frac{1}{2m}(2kv^2 + 2L + \frac{vK^2}{r}) \qquad (9)$$

In other words, the regenerating energy becomes the maximum.

According to Eq. (5), in the case of the following, the acceleration and deceleration efficiencies can be calculated.

$$P = \frac{r}{K^2}(kv^2 + L + ma)^2 + v(kv^2 + L) + mva = Rr + Pv + Pg$$

$$\begin{array}{lll} \text{loos by motor} & Pr = \frac{r}{K^2}(kv^2 + L + ma)^2 & \\ \text{loss by travel resistance} & Pv = v(kv^2 + L) & (10) \\ \text{energy obtained} & Pg = mva & \end{array}$$

Efficiency, $\eta_{a,}$ at the time of acceleration in the following equation indicates how (most efficiently) the input electricity contributes to the acceleration of vehicles.

$$\eta_a = \frac{Pg}{P} = \frac{Pg}{Pr + Pv + Pg} \qquad (11)$$

Efficiency, $\eta_{d,}$ of deceleration during travel in the following equation shows how effectively the input electricity is regenerated to a power-supply unit.

$$\eta_d = \frac{P}{Pg} = \frac{Pr + Pv + Pg}{Pg} \qquad (12)$$

When Eq. (10) and Eq. (11) are differentiated by an accelerated velocity, a, and the (equation) value becomes 0, the efficiency of the accelerated velocity becomes the maximum at the invariable, K, at a velocity, v, .

Here, K, is $K = \frac{K_a \phi N}{R}$ according to equations (3) and (4), and thus it is possible to change, K, by changing the reduction ratio, N, or the flux, φ, . The, a, in the equation is the function of, K, and , v, . When a transformation is made and the accelerated velocity, a, is on the horizontal axis and, K, on the vertical axis, it is possible to calculate the most efficient and comfortable, K, in order to have (any) necessary acceleration (in a case where acceleration is needed) at a velocity, v, .

4. DETERMINATION OF THE ARMATURE RESISTANCE

The relationship between the revolution of the DC motor, f[rps], and the back electromotive force is expressed by the following equation using the constant value of, K_e, .

$$E = K_e \cdot f \tag{13}$$

The value, K_e, is named the back electromotive force factor. The revolution of the motor, f, is obtained as a function of the input voltage, V, and the armature resistance, r, as,

$$E = V - Ir \tag{14}$$

From Eq. (13),

$$f = \frac{V - Ir}{K_e} \tag{15}$$

From the motor characteristics shown in Figure 2, K_e, is obtained using the revolution, f = 30 [rps], for the unload condition I = 0 [A], V = 24 [V], using Eq. (15).

$$K_e = \frac{24}{30} = 0.8 \tag{16}$$

The load current, I, used for this study as shown in Fig. 4 is almost 20-25 [A]. In such a region, the cupper loss, I^2r, is dominant. The input power, P, to the motor is expressed as, 24 I [W], when the motor voltage, I = 24 [V], . The motor efficiency, ς [%], appeared in Fig. 3 is calculated and shown by the ratio of the product of the motor torque and the revolution to the input power, P, . The armature resistance, r, is expressed as follows.

$$r = 24I(1 - 0.01\eta)/I^2 = 24(1 - 0.01\eta)/I \tag{17}$$

Using Eq. (17), the values of the efficiency, ς, at the current, I, of 20, 25, and, 30 [A], are 90, 88, and , 86 [%], respectively. The armature resistance, r, is obtained in average as,

$$r = 0.11 \ [\Omega] \tag{18}$$

4.1 Determination of K

When the car is driven by a gear-less system (the direct drive system), the velocity of the car is expressed using the motor revolution, f[rps], and the radius of the tire, R [m], as,

$$v = 2\pi R \cdot f \tag{19}$$

The relationship between the back electromotive force and the velocity of the ca ris expressed using Eq. (13) as,

$$E = \frac{K_e}{2\pi R} v = Kv \tag{20}$$

When the radius of the tire is 0.17[m], K, is obtained as,

$K = 0.72$

By the use of, r, K_e, and, K, obtained above equations, the change of the velocity of the car, a, to the input power, P, can be calculated based on Eq. (6).

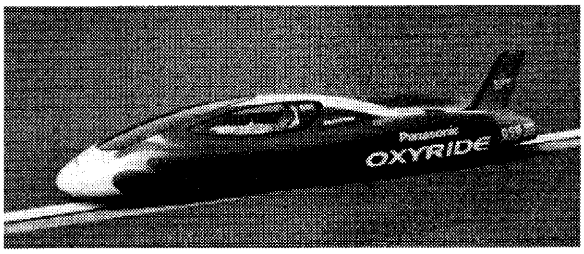

Fig. 1 Pictures of EV used for driving

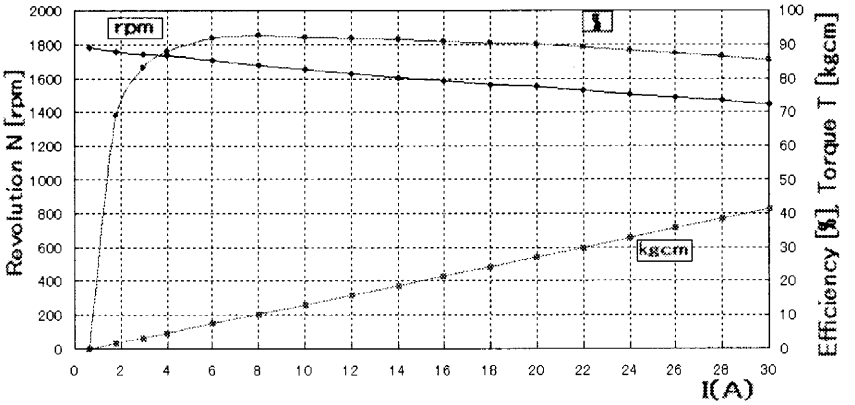

Fig. 2 Characteristics of the in-wheel motor which was used (Two motors were used for this EV) [Ashida et al., 2007]

Table 1 Parameters of a vehicle used for analysis [Ashida et al., 2007]

Body size (L-W-H mm)	3300 - 780 - 510	Air-resistance coefficient (Cd)	0.13
Frontal projected area of vehicle: A(m^2)	0.34	Rolling-resistance coefficient (μ)	0.002
Weight of vehicle (kg)	38	Motor average efficiency (%)	91
Total weight of vehicle (kg)	88	Drive-train transmission efficiency (%)	100
Diameter of a tire (cm)	34.0	Motor and drive wheel	DD motor rear 2-tire drive

Table 2 The load parameters of the EV used at 100 km/h [Ashida et al., 2007]

Rolling-resistance (N)	2.45
Air-resistance (N)	21.32
Acceleration resistance (N)	47.22
Total running resistance (N)	70.99

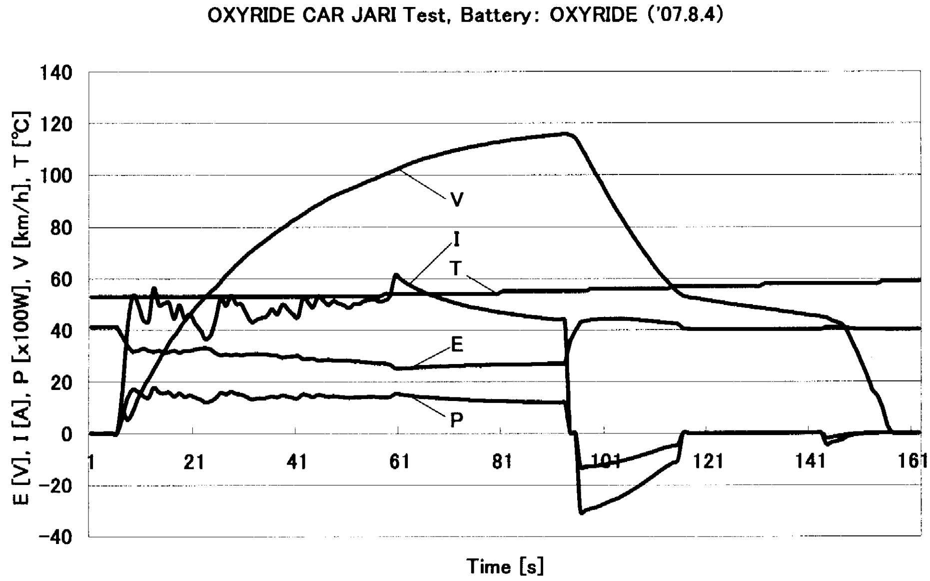

Fig. 3 Measured parameters of EV used for the calculation [Ashida and Minami, 2007, Ashida et al., 2007]

4.2 Examples of the velocity profile of the car obtained from the input power, P

If equation (6) is used, the relationship of the velocity profile which is predicted by inputting electricity and calculations is computed, and comparisons with actual measurement values become possible. The data used here is data obtained from a project to achieve a velocity of 100 km/h with a car powered by size-AA dry-cell batteries with a person on board. Figure 3 shows its time change. Table 1 shows the parameters of the EV used for the evaluation of this calculation. The load parameters at 100 km/h (including travel-resistance and air-resistance) of the EV shown in Table 2. Figure 1 is a picture of the EV used for the evaluation.

By using the electricity input to the EV, acceleration, a, is calculated based on equation (6). Veloc-

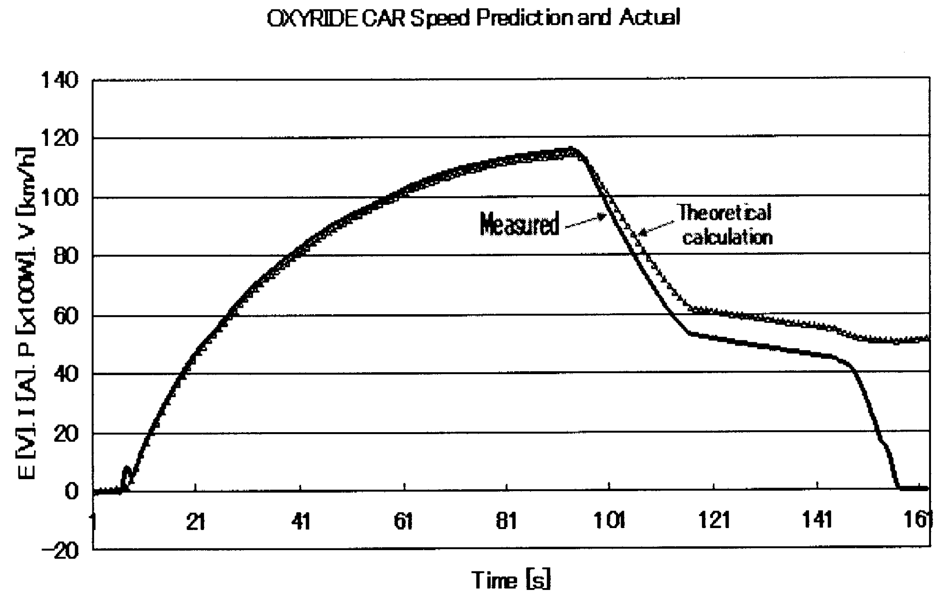

Fig. 4 The velocity profiles of the actually measured and the theoretically predicted

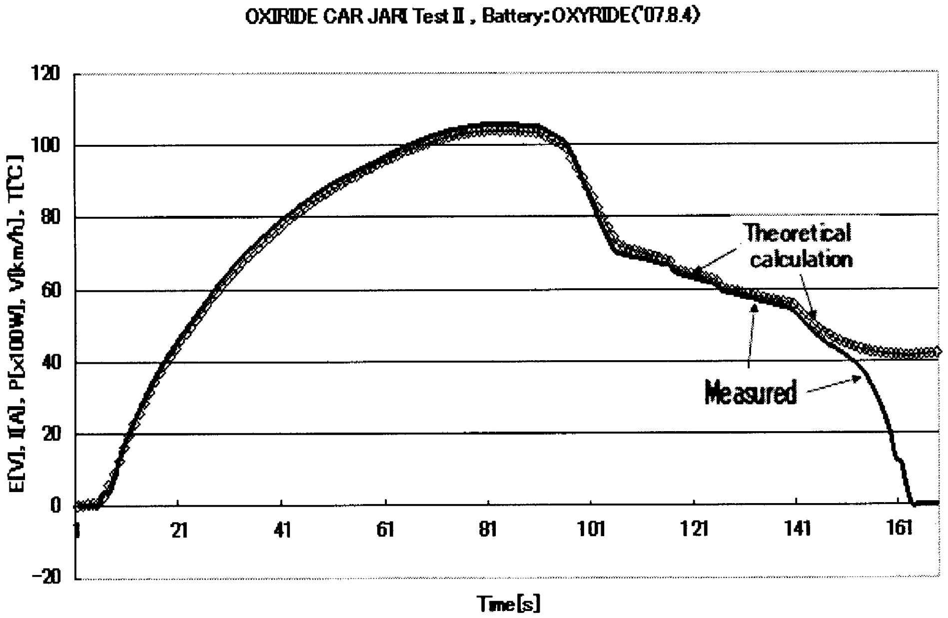

Fig. 5 The other velocity profiles of the actually measured and the theoretically predicted

ity, v, after the shot time, Dt, is given in the following equation.

$$v' = v + a\Delta t \tag{21}$$

Equivalent armature resistance, r, is obtained based on the table of motor characteristics in Figure 2. Figure 4 shows the velocity profile obtained by the calculations and the actual measurement values. These show excellent conformability.

But there exists a difference at the deceleration period. The difference shows the thermal loss caused by the mechanical brake. An EV's deceleration processes can be recaptured from the input power, P, obtained by regenerative breaking. This is shown clearly by the fact that the actual measurement values and calculation values are conformable, as Figure 5 shows, for driving in which velocity is reduced only by regenerative breaking till just before stopping (about at 150 seconds). In this case, the use of mechanical breaking after 150 seconds, which is just before stopping, is clear.

5. DISCUSSION AND CONCLUSION

Figure 4 and Figure 5 show the relationship between accelerated velocity and velocity. It is shown here that the calculation of velocity changes is possible by giving the first velocity and the input electricity at the time. The total travel distance can be obtained from the integration of velocity at the end. By giving the total electric energy W [J] which is applied to the vehicle, the possible time for travel or distance related to any velocity profile, which includes distance and acceleration and deceleration velocities, can be calculated by using this equation. Alternatively, by giving a velocity profile at the beginning, it is possible to calculate how much electric energy, W, or how many batteries on board are necessary when a vehicle runs for an indicated length of time. It is also possible to run a simulation of the strength of the magnetic field given to a motor or the optimization of the reduction ratio in order to have an effective improvement in fuel consumption. It can be pointed out that EV's are very convenient to perform such analysis, because the value of the eclectic input power can be obtained easily by a current and the voltage to the motor on board. The necessity of introducing such kind of thought is important for the EV's because of limited driving distance.

In conclusion, general equations for obtaining a velocity profile are formulated by giving time changes of electricity input power, P, in a vehicle. By applying the equations to actual data and by showing that the theoretical predicted values and the actual measurement values of the velocity profile conform well, it can be shown that the equations formulated in this paper have validity. This method is considered useful in order to obtain performance assessments of electric vehicle in the future.

References

Ashida, T., S. Minami, H. Fujita, A. Saihara, E. Uchiyama, and M. Otani, Designing Engines of Compact Battery-Cars for the Target Velocity of 100 km/h using Nickel-system primary-cells (Oxyride Dry-cell batteries) Part 1: Optimization of the body, *Journal of Asian Electric Vehicles*, Vol. 5, No. 2, 2007.

Ashida, T., and S. Minami, Designing Compact Battery-car for Which the Target Velocity is 100km/h Using Nickel-system Primary-cells (Oxyride Dry-cell Batteries) Part 2: Battery System Optimization, *Journal of Asian Electric Vehicles*, Vol. 5, No. 2, 2007.

Chan, C. C., and K. T., Chau, *Modern Electric Vehicle Technology*, Oxford Science Publications, 2001.

Nasukawa, K., Y. Miyashita, and M. Shiokawa, *Cruising Performance and the Testing Method of Automobiles* (in Japanese), Sankaido, 1993.

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